

## Exercise 55

A certain bacterium grows in culture in a circular region. The radius of the circle, measured in centimeters, is given by  $r(t) = 6 - \left[ \frac{5}{t^2 + 1} \right]$ , where  $t$  is time measured in hours since a circle of a 1-cm radius of the bacterium was put into the culture.

- Express the area of the bacteria as a function of time.
  - Find the exact and approximate area of the bacterial culture in 3 hours.
  - Express the circumference of the bacteria as a function of time.
  - Find the exact and approximate circumference of the bacteria in 3 hours.
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### Solution

#### Part (a)

Since the bacteria grows in a circle, the area (in square centimeters) is

$$A(t) = \pi[r(t)]^2 = \pi \left( 6 - \frac{5}{t^2 + 1} \right)^2.$$

#### Part (b)

In 3 hours the area is

$$A(3) = \pi \left( 6 - \frac{5}{3^2 + 1} \right)^2 = \pi \left( 6 - \frac{5}{10} \right)^2 = \pi \left( \frac{11}{2} \right)^2 = \frac{121\pi}{4} \text{ cm}^2 \approx 95.0 \text{ cm}^2.$$

#### Part (c)

Since the bacteria grows in a circle, the circumference (in centimeters) is

$$C(t) = 2\pi r(t) = 2\pi \left( 6 - \frac{5}{t^2 + 1} \right).$$

#### Part (d)

In 3 hours the circumference is

$$C(3) = 2\pi \left( 6 - \frac{5}{3^2 + 1} \right) = 2\pi \left( 6 - \frac{5}{10} \right) = 2\pi \left( \frac{11}{2} \right) = 11\pi \text{ cm} \approx 34.6 \text{ cm}.$$